Logistics / Warm-Up

Attendance: https://forms.gle/iMXPHXs6xW4qn5Ht8

- Optional Check-Ins this Sunday
  - Can discuss Midterm 1, and/or strategies for the rest of the quarter
  - Sign up here: https://calendly.com/esierra-stanford/103a-check-in
  - All check-ins on Zoom
  - Email/Slack me if the available times do not work for you, we can schedule another time!
- Preference form for Midterm 2 Review
  - https://forms.gle/2SavQC1fhe2Gte2QA

Work on Problem 1 (a) with your table!

https://forms.gle/iMXPHXs6xW4qn5Ht8

Midterm 1 Reflection

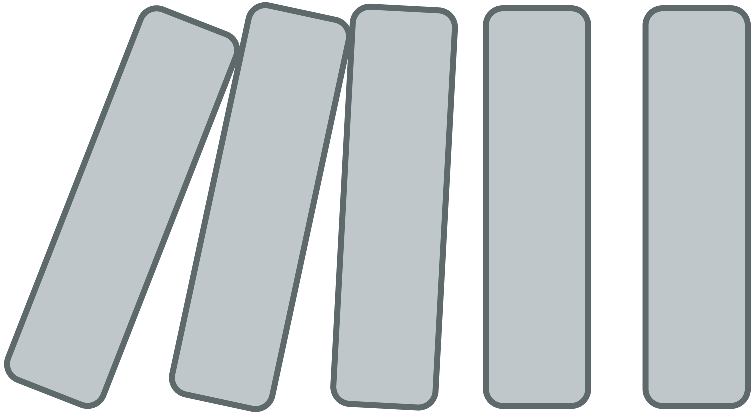
- Things to remember about the midterm
  - Going from proofs on PSETs to exams is difficult
  - Most of your grade (70%+ of it) is still in your hands. There is time to improve!
  - Exams do not reflect whether you belong in CS
  - I know it's a tough time of the quarter, you are not alone
- Fill out this anonymous feedback form about the exam / course in general for me / the course to improve!

https://forms.gle/Z5gGCz87iafibXiF9

Any Lingering Graph Questions?

- Graph slides: https://web.stanford.edu/class/archive/cs/cs103ace/cs103ace.1256/materials/Week%205.pdf
- Graph Worksheet: https://web.stanford.edu/class/archive/cs/cs103ace/cs103ace.1256/materials/CS103A_Week5_Probs.pdf

Induction: A real-world comparison



How do we know all the dominoes fall?
1st domino falls
If some domino falls, so does the next

Induction

To prove this statement:

For any natural number n,
$P(n)$ is true

We can use these steps:

1. **Base case:** Show that $P(0)$ is true.
2. **Inductive step:** Show that, for any natural number k,
   if $P(k)$ is true (the **inductive hypothesis**), then $P(k + 1)$ is true

Induction

To prove this statement:

For any natural number n,
$P(n)$ is true

We can use these steps:

1. **Base case:** Show that $P(0)$ is true.
2. **Inductive step:** Pick any natural number k.
   Assume $P(k)$ is true (the **inductive hypothesis**).
   Show that $P(k + 1)$ is true.

Important tips for choosing P(n)

- P(n) should be a predicate: a statement that can be true or false
  - P(n) is not a number
  - P(n) can be an equation itself, but shouldn't be used in equations
- You should be able to plug in a number for n
  - Try crossing out every "n" and replacing it with something like 0 or 103
  - P(n) should not introduce n as a new variable. Treat n like it already has a specific value!

Problem 1. Induction Walkthrough

1. **Restate the theorem with a predicate** P(n).
    a. Often: the exact theorem, crossing out “for all n…”
2. State the **base case** (show P(__) is true) and show it.
3. State the **inductive hypothesis** (pick a k and assume P(k) is true)
4. State the **inductive step goal** (show P(k + __) is true) and show it.
5. **Conclude** that P(n) is true for all natural numbers!

Refreshing on universal vs. existential

	a universal statement “for all unicorns u that are abc, u is xyz.”	**an existential statement** “there is a unicorn u that is abc and xyz.”
assuming		
proving		

Refreshing on universal vs. existential

	a universal statement “for all unicorns u that are abc, u is xyz.”	**an existential statement** “there is a unicorn u that is abc and xyz.”
assuming	Don’t do anything! If you find a unicorn that is abc, you can say that unicorn is xyz.	You can introduce a unicorn into your proof that is both xyz and abc.
proving	Ask the reader to pick a unicorn u that is abc. Show that u is xyz.	You need to come up with a specific value. Then, show that value/object is a unicorn, abc, and xyz.

Building up/down/neither: check the predicate

In induction, the overall theorem is always universal!

"Equation" P(k): something = something

Start with one side of P(k + 1), get to the other

"Build up" P(k): There exists... → P(k + 1): There exists...

Start with the thing we know exists from P(k)

"Build down" P(k): For all... → P(k + 1): For all...

Ask the reader to pick the subject of P(k+1)

Complete Induction Template

1. **Restate the theorem with a predicate** P(n).
2. State the **base case** (show P(__) is true) and show it.
3. State the **inductive hypothesis** (pick a k and assume **P(__), ..., P(k) are all true**)
   a. This assumption is the key to complete induction!
4. State the **inductive step goal** (show P(k + 1) is true) and show it.
5. **Conclude** that P(n) is true for all natural numbers!